MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2017/2018

DEM5028 – ENGINEERING MATHEMATICS 2
(Group: E17)

7 MARCH 2018
9.00 a.m. – 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of **4** pages (**2** pages with **4** questions and **2** pages of appendix.
2. Answer **ALL** questions. All necessary working steps must be shown.
3. Write all your answers in the answer booklet provided.

QUESTION 1 [25 MARKS]

a. Using suitable integration method, evaluate the following integrals.

i) $\int_0^1 x^2(x^3+3)^3 dx$

[6 marks]

ii) $\int 3xe^{2x} dx$

[7 marks]

b. Find the volume of the enclosed region bounded by $y = x-1$ and $x^2 + y = 1$ that rotate at $y = -4$.

[7 marks]

c. Find the area of the region bounded by the curves $y = x^2 + 2$, and $x + y = 2$.

[5 marks]

QUESTION 2 [25 MARKS]

a. Find the solution of the differential equation $y\dfrac{dy}{dx} = 2x$.

[4 marks]

b. For the differential equation of $\dfrac{dy}{dx} + y = e^x$, find the final solution that satisfy the condition of $y(0) = 1$.

[9 marks]

c. Test either the series of $\sum_{n=1}^{\infty} \dfrac{8\sqrt{n}}{n^3}$ is convergence or divergence. (Must state the test used).

[3 marks]

d. Consider the power series $\sum_{n=1}^{\infty} \dfrac{nx^n}{3^n}$

i) Find the radius of convergence.

[4 marks]

ii) Find the interval of convergence.

[5 marks]

Continued...

QUESTION 3 [25 MARKS]

a. Consider vector **a** = <–2, 2, –2> and **b** = <1, –3, –3>, find

   i) vector **b** x **a.**

[4 marks]

   ii) the angle between **a** and **b**.

[4 marks]

   iii) **b.(a** + 2**b)**

[6 marks]

b. Find the symmetric and parametric equations of the line that goes through the points $P(1, 2, 4)$ and $Q(3, -1, 6)$.

[6 marks]

c. Find the equation of the plane through the point (–2, 8, 10) and perpendicular to the line $x = 1 + t, y = 2t, z = 4 - 3t$.

[5 marks]

QUESTION 4 [25 MARKS]

a. For $f(x, y, z) = x^3 z - 3xy^2 - (yz)^3$, find all its first partial derivatives.

[3 marks]

b. Given $f(x, y) = x^2 + \frac{1}{3}y^3 - 2xy - 3y$.

   i) Find the critical point(s) of the function.

[4 marks]

   ii) Determine whether the critical point(s) is a maximum, minimum or saddle point.

[8 marks]

c. A lamina occupies a region between $x = -1$, $x = 1$, $y = 0$ and $y = 1$ has a density of $\rho(x, y) = x^2$.

   i) Find its mass

[2 marks]

   ii) Find its center of mass

[8 marks]

End of page.

APPENDIX I: Formulae

Integration of common functions

$\int x^n dx = \frac{x^{n+1}}{n+1} + C$ $\int \frac{1}{x} dx = \ln|x| + C$ $\int e^x dx = e^x + C$ $\int a^x dx = \frac{a^x}{\ln a} + C$

$\int \sin x dx = -\cos x + C$ $\int \cos x \, dx = \sin x + C$ $\int \sec^2 x \, dx = \tan x + C$ $\int \sec x \tan x dx = \sec x + C$

$\int \csc^2 x dx = -\cot x + C$ $\int \csc x \cot x dx = -\csc x + C$

Inverse Trigonometry

$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$$

$$\int \frac{1}{x^2+1} dx = \tan^{-1} x + C$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$
$$1 + \cot^2 x = \csc^2 x$$
$$1 + \tan^2 x = \sec^2 x$$

Integration by parts

$$\int u dv = uv - \int v du$$

Areas Between Curves

$$A = \int_a^b [f(x) - g(x)] dx$$

$$A = \int_c^d [w(y) - v(y)] dy$$

Volume by Washer

$$V = \int_a^b \pi\left([f(x)]^2 - [g(x)]^2\right) dx$$

$$V = \int_c^d \pi\left([w(y)]^2 - [v(y)]^2\right) dy$$

Volume by Cylindrical Shells

$$V = \int_a^b 2\pi x\left(f(x) - g(x)\right) dx$$

$$V = \int_c^d 2\pi y\left(w(y) - v(y)\right) dy$$

Linear Differential Equations:

$$\frac{dy}{dx} + p(x)y = q(x); \ \mu y = \int \mu q(x) dx \Rightarrow y = \frac{1}{\mu}\int \mu q(x) dx, \quad \text{where } \mu = e^{\int p(x)dx}$$

Divergence Test	If $\lim_{n\to\infty} a_n \neq 0$, then $\sum a_n$ diverges.
p-series	The p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if p > 1 and divergent if $p \leq 1$.
Limit Comparison Test	Let $\sum a_n$ and $\sum b_n$ be series with positive terms such that $\lim_{n\to\infty} \frac{a_n}{b_n} = c$ If $0 < c < \infty$, then both series converge or both diverge.
Alternating Series Test	If the alternating series $\sum_{n=1}^{\infty} (-1)^{n-1} b_n = b_1 - b_2 + b_3 - b_4 + \ldots \quad b_n > 0$ Satisfies : i. $b_{n+1} \leq b_n$ for all n ii. $\lim_{n\to\infty} b_n = 0$ then the series is convergent
Ratio Test	Let $\sum a_n$ be a series with nonzero terms such that $L = \lim_{n\to\infty} \frac{\lvert a_{n+1}\rvert}{\lvert a_n\rvert}$ a. Series converges absolutely if $L < 1$ b. Series diverges if $L > 1$ or $L = \infty$ c. No conclusion if $L = 1$

Quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Vector

The length of the vector **a** $=< a_1, a_2, a_3 >$ is $|a| = \sqrt{a_1^{\,2} + a_2^{\,2} + a_3^{\,2}}$.

If θ is the angle between the vector **a** and **b**, then **a.b** $= |\mathbf{a}||\mathbf{b}|\cos\theta$ & $|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}|$ $\sin\theta$

Cross Product

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} i - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} j + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} k$$

Equation of Line

Vector equation: $r = r_0 + tv$ Parametric equation: $x=x_0+at,\ y=y_0+bt,\ z=z_0+ct$

Equation of Plane $< a, b, c >< x - x_0, y - y_0, z - z_0 >= 0$

The Chain Rule

Suppose that $z = f(x, y)$, where $x = g(t)$ and $y = h(t)$ $\Rightarrow$ $\dfrac{dz}{dt} = \dfrac{\partial f}{\partial x}\dfrac{dx}{dt} + \dfrac{\partial f}{\partial y}\dfrac{dy}{dt}$

Second Derivatives Test

Suppose that $f_x(a,b) = 0$ and $f_y(a,b) = 0$ [that is, (a, b) is a critical point of f]. Let $D = D(a,b) = f_{xx}(a,b) f_{yy}(a,b) - [f_{xy}(a,b)]^2$

a. If $D > 0$ and $f_{xx}(a,b) > 0$, then $f(a, b)$ is a local minimum.
b. If $D > 0$ and $f_{xx}(a,b) < 0$, then $f(a, b)$ is a local maximum
c. If $D < 0$, then $f(a, b)$ is a saddle point.

Moments and Centers of Mass

The **moment** about the ***x*-axis:**

$$M_x = \iint_D y\rho(x, y)\,dA$$

The **moment** about the **y-axis:**

$$M_y = \iint_D x\rho(x, y)\,dA$$

The coordinates $(\bar{x}, \bar{y})$ of the center of mass:

$$\bar{x} = \frac{M_y}{m} = \frac{1}{m}\iint_D x\rho(x, y)\,dA \qquad \bar{y} = \frac{M_x}{m} = \frac{1}{m}\iint_D y\rho(x, y)\,dA$$

Where the mass:

$$m = \iint_D \rho(x, y)\,dA$$

Triple Integrals: $\iiint_B f(x, y, z)\,dV = \int_r^s \int_c^d \int_a^b f(x, y, z)\,dx\,dy\,dz$